

Spaces with a σ -Point Finite Base*

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Abstract In this paper it is shown that spaces with a σ -point finite base can be characterized by the images of metric spaces under certain maps

Key words σ -point finite base, open map, compact map, continuous map.

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All spaces considered in this paper are T_1 .

In 1963, S Hanai^[1] proved that a space has a σ -point finite base if and only if it is an image of a metric space under an open, compact and continuous map. L. F. Liu^[2] also obtained the same result. The first result in this paper is to point out a counter-example to show the result mentioned above is not true. Next we introduce the concept of inductive open, weak compact and continuous maps, and prove that a space has a σ -point finite base if and only if it is an image of a metric space under an inductive open, weak compact and continuous map.

First of all, we describe a relationship between spaces with a σ -point finite base and images of metric spaces under open, compact and continuous maps. The following Lemma is well known, cf. [3, 4].

Lemma A T_2 space is a perfect space with a σ -point finite base if and only if it is an image of a metric space under an open, compact and continuous map.

Example There is a regular space X with a σ -point finite base which is not a perfect space.

H. H. Corson and E. Michael^[5] has shown that there is a non-perfect, regular space X with a σ -point finite base (Example 6.4 in [5]). By Lemma, X is not an image of a metric space under an open, compact and continuous map. Thus Theorem 2 in [1] and Theorem 1 in [2] are wrong. The mistake of the proof of Theorem 1 in [2] is that the map f constructed in the proof of the necessity can not be a compact map because $f^{-1}(x)$ is only a subset of the compact subspace C_x of the metric space $\prod_{N \times \mathbb{N}} \mathbb{N}$. Hence the following question can be raised: By means of what maps can the relationship between metric spaces and spaces with a

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σ point finite base be established? Next we shall answer this question by the concept of inductive open, weak compact and continuous maps

Definition 1 Let Z be a space. A subset Y of Z is said to be weak compact in Z if the closure of Y in Z is compact in Z .

Definition 2 Let g be a map from a space Z onto a space X . g is said to be inductive open, weak compact and continuous if there is a subspace S of Z such that $g|_S$ is an open and continuous map from S onto X , and $g^{-1}(x) \cap S$ is weak compact in Z for each $x \in X$.

Theorem A A space has a σ point finite base if and only if it is an image of a metric space under an inductive open, weak compact and continuous map.

Proof Necessity. By the proof of the necessity of Theorem 1 in [2], there are a metric space $Z (= \prod_{i \in \mathbb{N}} A_i)$, a subspace S of Z and an open and continuous map f from S onto X such that there is a compact subspace C_x of Z with $f^{-1}(x) \subset C_x$ for each $x \in X$. Take a point $x_0 \in X$. We define a map g from Z onto X by

$$g(z) = \begin{cases} f(z), & z \in S, \\ x_0, & z \in Z \setminus S. \end{cases}$$

It is easy to check that g is an inductive open, weak compact and continuous map from Z onto X .

Sufficiency. Let g be an inductive open, weak compact and continuous map from a metric space Z onto X . Then there is a subspace S of Z such that $g|_S$ is an open, continuous map from S onto X , and $g^{-1}(x) \cap S$ is weak compact in Z for each $x \in X$. Let $f = g|_S$. Since Z is metric, it has a σ -locally finite base \mathbf{B} by the classical Nagata-Smirnov metrization theorem. Denote \mathbf{B} by $\{\mathbf{B}_n; n \in \mathbb{N}\}$, where \mathbf{B}_n is locally finite in Z for each $n \in \mathbb{N}$. Put

$$\mathbf{P} = \{\mathbf{P}_n; n \in \mathbb{N}\},$$

where $\mathbf{P}_n = \{f(B \cap S); B \in \mathbf{B}_n\}$ for each $n \in \mathbb{N}$, then \mathbf{P} is a base of X because f is an open and continuous map from S onto X . For each $x \in X$, $f^{-1}(x)$ is compact in Z , then

$$\overline{\{f^{-1}(x) \cap B; B \in \mathbf{B}_n\}}$$

is finite by the local finiteness of \mathbf{B}_n , thus

$$\overline{\{f^{-1}(x) \cap B; B \in S \cap \mathbf{B}_n\}}$$

is finite. Since $f^{-1}(x)$ is closed in S ,

$$\overline{f^{-1}(x) \cap S} = f^{-1}(x),$$

hence $\{f^{-1}(x) \cap B; B \in S \cap \mathbf{B}_n\}$ is finite, therefore \mathbf{P}_n is point finite at point x . So, \mathbf{P} is a σ point finite base for X , i.e., the space X has a σ point finite base

References

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具有 σ 点有限基的空间

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摘 要

本文纠正了 Hanai 等用开紧连续映射刻画具有 σ 点有限基的空间的一个错误, 定义了诱导开的弱紧连续映射建立度量空间与具有 σ 点有限基空间的映射联系