Studia Scientiarum Mathematicarum Hungarica 51 (3), 326–334 (2014)
DOI: 10.1556/SScMath.51.2014.3.1286

A NOTE ON THE CONTINUITY OF THE INVERSE IN PARATOPOLOGICAL GROUPS*

LI-HONG XIE[1] and SHOU LIN[2]

[1] School of Mathematics and Computational Science, Wuyi University, Jiangmen 529020, P. R. China
e-mail: xielihong2011@aliyun.com

[2] Institute of Mathematics, Ningde Normal University, Ningde 352100, P. R. China
e-mails: shoulin60@163.com, shoulin60@aliyun.com

Communicated by I. Juhász

(Received November 15, 2013; accepted May 20, 2014)

Abstract

The problem when a paratopolgical group (or semitopological group) is a topological group is interesting and important. In this paper, we continue to study this problem. It mainly shows that: (1) Let G be a paratopological group and put $\tau = wHs(G)$; then G is a topological group if G is a P_τ-space; (2) every co-locally countably compact paratopological group G with $wHs(G) \leqq \omega$ is a topological group; (3) every co-locally compact paratopological group is a topological group; (4) each 2-pseudocompact paratopological group G with $wHs(G) \leqq \omega$ is a topological group. These results improve some results in [11, 13].

1. Introduction

With an algebraic structure, say a group, and a topology on it, one can require distinct types of relation between them. If, for example, the multiplication in the group is jointly (separately) continuous, then this object is called a *paratopological* (*semitopological*) *group.* If in addition the inversion in the paratopological group is continuous, then it is called a *topological group.*

2010 *Mathematics Subject Classification.* Primary 54H11, 54H13, 54H99.
Key words and phrases. Paratopological groups, topological groups, weakly Hausdorff number, P-spaces, co-locally countably compact paratopological groups, 2-pseudocompact paratopological groups.

*The project is supported by the Science Foundation for Young Teachers of Wuyi University (No. 2013zk03) and the NSFC (Nos. 11271262, 11201414).

The question of finding topological properties on a topological semigroup (or paratopological group) which imply that it is in fact a topological group has many precedents in the literature. By the celebrated theorem of Numakura in [10], every compact Hausdorff topological semigroup with two-sided cancellation is a topological group. In particular, compact Hausdorff paratopological groups are topological groups. The latter fact has been considerably generalized by Ellis [7], Grant [8], Brand [6], Bouziad [5], Bokalo and Guran [4], Romaguera and Sanchis [15], Kenderov et al. [9], and some others.

The study of automatic continuity in paratopological groups was done by Reznichenko in [14] where he proved that every completely regular pseudocompact paratopological group G is a topological group, i.e., the inversion in G is continuous. In [2], Arhangel'skiĭ and Reznichenko extended this result to regular pseudocompact paratopological groups. Some other results can be found in [1, 11, 13].

In this paper we continue to study the problem when a paratopological group is a topological group. It mainly improves some results in [11,13]. The paper is organized as follows. In Section 2, we define an important cardinal function in a paratopological group G, which is called weakly Hausdorrf number, denoted by $wHs(G)$. This cardinal function plays an important role in our study. Some simple results on this cardinal function are established and we also show that: Let G be a paratopological group and put $\tau = wHs(G)$; then G is a topological group if G is a P_τ-space (Theorem 2.7) in this section. In Section 3, we mainly study co-locally countably compact and 2-pseudocompact paratopological groups. It mainly shows that: (1) Every co-locally countably compact paratopological group G with $wHs(G) \leqq \omega$ is a topological group (Theorem 3.1); (2) every co-locally compact paratopological group G is a topological group (Theorem 3.6); (3) each 2-pseudocompact paratopological group G with $wHs(G) \leqq \omega$ is a topological group (Theorem 3.8).

No separation restrictions on the topological spaces considered in this paper are imposed unless we mention them explicitly. Below $\psi(X)$, $\chi(X)$ and $l(X)$ denote the pseudocharacter, character and Lindelöf number of a space X, respectively.

2. The weakly Hausdorff number of paratopological groups and P-spaces

Recall that for a Hausdorff paratopological group G with the identity e the *Hausdorff number* [17] of G, denoted by $Hs(G)$, is the minimum cardinal number κ such that for every neighborhood U of e in G, there exists a family γ of neighborhoods of e such that $\bigcap_{V\in\gamma} VV^{-1} \subset U$ and $|\gamma| \leqq \kappa$. This observation gives rise to the following cardinal invariant.

For a semitopological group G with the identity e the *weakly Hausdorff number* of G, denoted by $wHs(G)$, is the minimum cardinal number κ such that for every neighborhood U of e in G there exists a family γ of neighborhoods of e such that $\bigcap_{V\in\gamma} V^{-1} \subset U$ and $|\gamma| \leqq \kappa$.

It is clear from the definition that G is a topological group if and only if G is a paratopological group with $wHs(G) = 1$. Evidently, $wHs(G)$ is infinite whenever G fails to be a topological group. For example, the Sorgenfrey line S satisfies $wHs(S) = \omega$. The proofs of the following two simple facts are left to the reader.

Proposition 2.1. *$wHs(G) \leqq Hs(G)$, for any Hausdorff paratopological group G.*

Proposition 2.2. *Every T_1 paratopological group G satisfies the inequalities $wHs(G) \leqq \psi(G) \leqq \chi(G)$.*

Proposition 2.3. *Let G be a T_1 paratopological group. Then the inequality $wHs(G) \leqq l\big(\overline{V^{-1}}\big)$ holds for every nonempty open set V of G.*

Proof. Since G is a paratopological group, it is enough to show that the inequality $wHs(G) \leqq l\big(\overline{V^{-1}}\big)$ holds for every open neighborhood V of the identity e in G. Take any open neighborhoods V, U of e and put $\tau = l\big(\overline{V^{-1}}\big)$. Clearly $l\big(\overline{V^{-1}} \setminus U\big) \leqq \tau$. Since G is a paratopological group, one can easily find a family $\gamma = \{W_\alpha : \alpha < \tau\}$ of open neighborhoods at e and a subset $A = \{a_\alpha : \alpha < \tau\} \subset \overline{V^{-1}} \setminus U$ such that $\overline{V^{-1}} \setminus U \subset \bigcup_{\alpha<\tau} a_\alpha W_\alpha$ and that $e \notin a_\alpha W_\alpha^2$ for each $\alpha < \tau$. Put $\delta = \gamma \cup \{V\}$. One can easily verify that $\bigcap_{W\in\delta} W^{-1} \subset U$. It implies that $wHs(G) \leqq \tau = l\big(\overline{V^{-1}}\big)$. □

According to Proposition 2.3 the following corollary is obvious, which was proved independently in [16].

Corollary 2.4. *The inequality $wHs(G) \leqq l(G)$ is valid for any T_1 paratopological group G.*

A group G with the identity e is called *periodic* if for each $x \in G$ there exists an integer $n \geqq 1$ such that $x^n = e$. Recall that a paratopological group G is called *topologically periodic* [4] if for each $x \in G$ and every neighborhood U of the identity, there exists an integer $n \geqq 1$ such that $x^n \in U$. Clearly, every periodic paratopological group is topologically periodic.

Indeed, Proposition 2.5 was proved in the proof of [4, Theorem 3]. For the sake of completeness, we give its proof.

Proposition 2.5. *For every topologically periodic paratopological group G satisfies the inequality $wHs(G) \leqq \omega$.*

Proof. Let U be an arbitrary neighborhood of the identity e in G and $\{V_i : i \in \omega\}$ a family of neighborhoods of e such that $V_0 = U$ and $V_{i+1}^2 \subset V_i$ for each $i \in \omega$.

We shall show that $F = \bigcap \{ V_i^{-1} : i \in \omega \} \subset U$, which implies that $wHs(G) \leqq \omega$. Let $x \in F$, then $x^{-1} \in V_i$ for each $i \in \omega$. Since G is topologically periodic, one can choose $n \in \omega \setminus \{0\}$ such that $x^n \in V_1$. If $n = 1$, then $x \in U$ is obvious. Thus we can assume that $n > 1$. Clearly, $(x^{-1})^{n-1} \in V_i^{n-1}$ for each $i \in \omega$. Now choose $i_0 \in \omega$ such that $V_{i_0}^{n-1} \subset V_1$. Then $x = x^n (x^{-1})^{n-1} \in V_1^2 \subset U$. □

Corollary 2.6. *The inequality $wHs(G) \leqq \omega$ holds for every periodic paratopological group G.*

Let τ be an infinitely cardinal number. Recall that a space X is a P_τ-*space* if for any family γ of open neighborhoods such that $|\gamma| \leqq \tau$ the set $\bigcap_{U \in \gamma} U$ is open in X. If $\tau = \omega$, then X is called a *P-space.*

Theorem 2.7. *Let G be a paratopological group and put $\tau = wHs(G)$. Then G is a topological group if G is a P_τ-space.*

Proof. Take any open neighborhood U of the identity e in G. Hence there exists a family γ of open neighborhoods at e such that $|\gamma| \leqq \tau$ and $\bigcap_{V \in \gamma} V^{-1} \subset U$. It implies that $\left(\bigcap_{V \in \gamma} V \right)^{-1} = \bigcap_{V \in \gamma} V^{-1} \subset U$. Since G is P_τ-space, the set $\bigcap_{V \in \gamma} V$ is open in G. Hence G is a topological group. □

Corollary 2.8. *Every topologically periodic (or periodic) paratopological group G, which is a P-space, is a topological group.*

Proof. The statement directly follows from Proposition 2.5 and Theorem 2.7. □

Lemma 2.9 ([8]). *If $V^2 \subset U$, where V and U are open neighborhoods of the identity of a paratopological group, then $\left(\overline{V^{-1}}\right)^{-1} \subset U$.*

Corollary 2.10. *Let G be a T_1 paratopological group and P-space, which satisfies one of the following conditions. Then G is a topological group.*

(1) *G is Lindelöf* [11, (vii) of Theorem 4]*;*

(2) *there exists a nonempty open set V in G such that V^{-1} is Lindelöf* [11, (viii) of Theorem 4].

Proof. According to Proposition 2.3 and Theorem 2.7, it is enough to prove that there exists an open neighborhood W of the identity e in G such that $\overline{W^{-1}}$ is Lindelöf. We can also assume that the open set V contains e by the homogeneity of G. Thus, it is enough to take an open neighborhood W of e such that $W^2 \subset V$. Then $\overline{W^{-1}} \subset V^{-1}$ by Lemma 2.9. Since V^{-1} is Lindelöf, so is $\overline{W^{-1}}$. □

3. Co-locally countably compact and 2-pseudocompact paratopological groups

Let $\mathcal{P}$ be a topological property and G a paratopological group. We say that G is *co-local* $\mathcal{P}$ (i.e., *conjugate-local* $\mathcal{P}$) if there exists a nonempty open set V in G such that $\overline{V^{-1}}$ has the property $\mathcal{P}$. The following theorem is one of our main results.

Theorem 3.1. *Every co-locally countably compact paratopological group G with $wHs(G) \leqq \omega$ is a topological group.*

Proof. Since G is a paratopological group, by the homogeneity of G we can assume that there exists an open neighborhood V of the identity e in G such that $\overline{V^{-1}}$ is countably compact. Take any open neighborhood U of e. Since $wHs(G) \leqq \omega$, one can find a family $\{W_i : i \in \omega\}$ of open neighborhoods of e such that $\bigcap_{i\in\omega} W_i^{-1} \subset U$. We can also assume that the family $\{W_i : i \in \omega\}$ satisfies $W_{i+1}^2 \subset W_i$ for each $i \in \omega$. By Lemma 2.9, we have $\bigcap_{i\in\omega} \overline{W_i^{-1}} = \bigcap_{i\in\omega} W_i^{-1} \subset U$. Since $\overline{V^{-1}} \setminus U$ is countably compact, there exists $i_0 \in \omega$ such that $W_{i0}^{-1} \cap (\overline{V^{-1}} \setminus U) = \emptyset$. Put $W = W_{i_0} \cap V$. Then it is obvious that W contains e such that $W^{-1} = W_{i_0}^{-1} \cap V^{-1} \subset U$, which implies that G is a topological group. □

Corollary 3.2. *Let G be a paratopological group, which satisfies one of the following conditions. Then G is a topological group.*

(1) *G is countably compact with $wHs(G) \leqq \omega$;*

(2) *G is T_1 countably compact with $\psi(G) \leqq \omega$* [11, (iv) of Theorem 4]*;*

(3) *G is a T_1-space and there exists a nonempty open set V in G such that V^{-1} is countably compact with $\psi(G) \leqq \omega$* [11, (v) of Theorem 4]*;*

(4) *G is countably compact and topologically periodic* [4, Theorem 3]*;*

(5) *G is topologically periodic and there exists a nonempty open set V in G such that V^{-1} is countably compact;*

(6) *G is T_1 compact* [11, (i) of Theorem 4]*;*

(7) *G is a T_1-space and there exists a nonempty open set V in G such that V^{-1} is compact* [11, (ii) of Theorem 4]*.*

Proof. It is obvious that (2) ⇒ (3), (4) ⇒ (5) and (6) ⇒ (7).

(1) The statement directly follows from Theorem 3.1.

(3) It follows from Proposition 2.2, Lemma 2.9, and Theorem 3.1.

(5) The statement directly follows from Proposition 2.5, Lemma 2.9 and Theorem 3.1.

(7) The statement directly follows from Proposition 2.3, Lemma 2.9 and Theorem 3.1. □

Let X be a space and A a subset of X. Recall that *countably compact at* A of X if each infinite subset B of A has an accumulation point x in X. X is *countably compact* if X is countably compact at itself and X is *countably pracompact* if X is countably compact at a dense subset of X. A space X is called *pseudocompact* if every continuous real-valued function on X is bounded. For those classes of spaces the following relations are well known. All reversions of the following do not hold.

countably compact $\Rightarrow$ countably pracompact $\Rightarrow$ pseudocompact.

REMARK 3.3. In view of the (3) of Corollary 3.2, Raghavan and Reilly [11] asked whether every T_1 pseudocompact paratopological group G with $\psi(G) \leqq \omega$ is a topological group. Indeed, there exists a Hausdorff first-countable and countably pracompact paratopological group G but G is not a topological group [13, Example 3], which implies that the condition "countably compact" in Theorem 3.1 cannot be replaced by "countably pracompact".

Indeed, the T_1-separation axiom can be eliminated from the (7) of Corollary 3.2. First we give a simple lemma which was proved in [12, Corollary 1.1].

LEMMA 3.4 ([12, Corollary 1.1]). *Let G be a paratopological group, H a compact subgroup of G and F a closed subset of G. Then FH and HF are closed.*

Recall that a nonempty subset M of a semigroup S is called *right ideal* in S if $MS \subset M$. A right ideal M of a semigroup S is minimal if the equality $M = N$ holds for any right ideal N in S whenever $N \subset M$. The following result is probably known.

LEMMA 3.5. *Every compact semigroup has a minimal closed right ideal.*

PROOF. Let S be a compact semigroup. Put $\mathcal{I} = \{M \subset S : M$ is a closed right ideal in $S\}$. Clearly, $S \in \mathcal{I}$. One can easily verify that $\bigcap \gamma$ is a right ideal for any family γ of right ideals in S such that $\bigcap \gamma \neq \emptyset$. Then applying the Zorn lemma to the family $\mathcal{I}$ ordered by inverse inclusion one can easily complete the proof. □

THEOREM 3.6. *Every co-locally compact paratopological group G is a topological group.*

PROOF. In this proof, denote by $\mathscr{N}^G$ all the open neighborhoods of the identity in the paratopological group G.

By the homogeneity of G one can take an element $V \in \mathscr{N}^G$ such that $\overline{V^{-1}}$ is compact in G. Put $B = \bigcap \mathscr{N}^G$. Then it is clear that B is a semigroup.

Put $B' = \overline{\{e\}} = \bigcap_{U \in \mathscr{N}^G} U^{-1} = B^{-1}$. Clearly, B' is a compact semigroup, since $\overline{\{e\}} \subset \overline{V^{-1}}$. From Lemma 3.5 it follows that there exists a minimal closed right ideal H in B'. For an arbitrary element $x \in H$, we have that $xH \subset HB' \subset H$. It is also clear that $xHB' = x(HB') \subset xH$, i.e., xH is a right ideal in B'. Since xH is closed in B' and $xH \subset H$, and H is a minimal right ideal in B', we conclude that $xH = H$ for each $x \in H$. In particular, $x^2H = H$ for each $x \in H$, whence it follows that $x^2y = x$ for some $y \in H$ and hence $x^{-1} = y \in H$. In its turn, this implies that $e \in H$, $H = B'$, and that B' is a group, In particular, $B = B'$. Since $B = \bigcap \mathscr{N}^G$ we see that $g^{-1}Bg \subset B$ for each $g \in G$. Hence B is a normal subgroup.

Denote by G/B the quotient group of G, and endow it with the quotient topology with respect to the canonical mapping $\pi : G \to G/B$. Since the set B' is closed and compact, we see that the quotient group G/B is a T_1-space and π is closed by Lemma 3.4. Since every quotient homomorphism in the class of paratopological group is open, $\pi(V) \in \mathscr{N}^{G/B}$ and such that $\big(\pi(V)\big)^{-1} = \pi\big(V^{-1}\big) \subset \pi\big(\overline{V^{-1}}\big)$. Hence the closure of $\big(\pi(V)\big)^{-1}$ (take in G/B) is compact in G/B. From the (7) of Corollary 3.2 it follows that G/B is a topological group.

Take any $U \in \mathscr{N}^G$. Then there exists $W_1 \in \mathscr{N}^G$ such that $W_1^2 \subset U$. Since π is open and G/B is a topological group, there exists $W_2 \in \mathscr{N}^{G/B}$ such that $W_2^{-1} \subset \pi(W_1)$. Hence $\big(\pi^{-1}(W_2)\big)^{-1} \subset \pi^{-1}\big(\pi(W_1)\big) = W_1B \subset W_1^2 \subset U$. It implies that G is a topological group. □

Corollary 3.7 ([18, Proposition 3.2]). *A compact paratopological group is a topological group.*

A sequence $\{U_n : n \in \omega\}$ of subsets of a space X is non-increasing if $U_n \supset U_{n+1}$ for each $n \in \omega$. A (not necessarily T_0) paratopological group G is *2-pseudocompact* [13] if $\bigcap_{i\in\omega} \overline{U_i^{-1}} \neq \emptyset$ for each non-increasing sequence $\{U_n : n \in \omega\}$ of nonempty open subsets of G. It is obvious that every countably compact paratopological group is 2-pseudocompact. Thus Theorem 3.8 (proved independently in [16]) improves (1) of Corollary 3.2.

Theorem 3.8. *Each* 2*-pseudocompact paratopological group* (G, τ) *with* $wHs(G) \leqq \omega$ *is a topological group.*

Proof. In this proof, denote by $\mathscr{N}^G$ all the open neighborhoods of the identity in a paratopological group G.

Take any $U \in \mathscr{N}^G$. Then there exists $W \in \mathscr{N}^G$ such that $W^2 \subset U$. By $wHs(G) \leqq \omega$, one can find a family $\gamma = \{V_i : i \in \omega\} \subset \mathscr{N}^G$ such that $\bigcap_{i\in\omega} V_i^{-1} \subset W$. Without loss of generality, we can assume that $V_{i+1}^2 \subset V_i$ for each $i \in \omega$. Hence we have $\overline{V_{i+1}^{-1}} \subset V_i^{-1}$ for each $i \in \omega$ by Lemma 2.9. Take W' as the closure of W in the paratopological group $\big(G, \tau^{-1}\big)$, where

$\tau^{-1} = \left\{ F^{-1} : F \in \tau \right\}$. Put $V_i' = V_i^{-1} \setminus W'$ for each $i \in \omega$. Then we have the following claim.

Claim: there exists $i_0 \in \omega$ such that $V_{i_0}^{-1} \subset W'$, which is equivalent to $V_{i_0}' = \emptyset$.

Assume to the contrary, i.e., there is no $i \in \omega$ such that $V_i' = \emptyset$, then the family $\{V_i' : i \in \omega\}$ is a sequence of decreasing nonempty open sets in $\left(G, \tau^{-1}\right)$. We shall prove that the family $\{V_i' : i \in \omega\}$ is local finite in (G, τ), which is a contradiction.

In fact, take any $x \in G$. Case 1: $x \in (G \setminus W)$. Since $\bigcap_{i\in\omega} \overline{V_i^{-1}} \subset W$, there exists $i_0 \in \omega$ such that $x \notin \overline{V_{i_0}^{-1}}$. Thus, $G \setminus \overline{V_{i_0}^{-1}}$ is an open neighborhood of x in (G, τ) such that $(G \setminus \overline{V_{i_0}^{-1}}) \cap V_n' = \emptyset$ for each $n > i_0$. Case 2: $x \in W$. Clearly, W is an open neighborhood of x in (G, τ) such that $W \cap V_n' = \emptyset$ for each $n \in \omega$. From cases 1 and 2 it follows that the family $\{V_i' : i \in \omega\}$ is local finite in (G, τ). It completes the proof of our claim.

From Lemma 2.9 and our claim it follows that $V_{i_0}^{-1} \subset W' \subset U$, which implies that (G, τ) is a topological group. $\square$

Corollary 3.9 ([13, Proposition 6]). *Each 2-pseudocompact paratopological group G with $\psi(G) \leqq \omega$ is a topological group.*

Proof. This directly follows from Proposition 2.2 and Theorem 3.8. $\square$

Corollary 3.10. *Each Hausdorff 2-pseudocompact paratopological group G with countable π-character is a compact metrizable topological group.*

Proof. Let $\mathscr{B}$ be a local base at the identity e of G and $\mathscr{C} = \{V_n \mid n \in \omega\}$ a local π-base at e. Take any $x \in G$ such that $x \neq e$. Since G is Hausdorff, there exists $U \in \mathscr{B}$ such that $x \notin UU^{-1}$. Thus there exists $n_0 \in \omega$ such that $V_{n_0} \subset U$, which implies that $x \notin UU^{-1} \supseteqq V_{n_0}V_{n_0}^{-1}$. Hence, $\{e\} = \bigcap_{n\in\omega} V_n V_n^{-1}$. It implies that $\psi(G) \leqq \omega$. It follows from Corollary 3.9 that G is a topological group. It is well known that every topological group with countable π-character is first countable [3, (a) of Proposition 5.2.6]. Thus G is metrizable [3, Theorem 3.3.12]. Since every 2-pseudocompact paratopological group is feebly compact [13, Proposition 2], G is compact. $\square$

Acknowledgement. The authors would like to express their sincere appreciation to the referee for his/her careful reading of the paper and valuable comments.

REFERENCES

[1] Alas, O. T. and Sanchis, M., Countably Compact Paratopological Groups, *Semigroup Forum.,* **74** (2007), 423–438.

[2] Arhangel'skiĭ, A. V. and Reznichenko, E. A., Paratopological and semitopological groups versus topological groups, *Topology Appl.,* **151** (2005), 107–119.

[3] Arhangel'shiĭ, A. V. and Tkachenko, M., *Topological Groups and Related Structures,* Atlantis Press, World Sci., 2008.

[4] Bokalo, B. and Guran, I., Sequentially compact Hausdorff cancellative semigroup is a topological group, *Mat. Stud.,* **6** (1996), 39–40.

[5] Bouziad, A., Every Čech-analytic Baire semitopological group is a topological group, *Proc. Am. Math. Soc.,* **124** (1996), 953–959.

[6] Brand, N., Another note on the continuity of the inverse, *Arch. Math.,* **39** (1982), 241–245.

[7] Ellis, R., A note on the continuity of the inverse, *Proc. Am. Math. Soc.,* **8** (1957), 372–373.

[8] Grant, D., Sequentially compact cancellative topological semigroups: some progress on the Wallace problem in "Papers on General Topology and Applications, Seventh Summer Conference at the University of Wisconsin" (Madison, 1991), *Annals of the New York Acad. Sci.,* **704** (1993), 150–154.

[9] Kenderov, P. S., Kortezov, I. S. and Moors, W. B., Topological games and topological groups, *Topology Appl.,* **109** (2001), 157–165.

[10] Numakura, K., On bicompact semigroups, *Math. J. Okayama Univ.,* **1** (1952), 99–108.

[11] Raghavan, T. G. and Reilly, I. L., On the continuity of group operations, *Indian J. Pure Appl. Math.,* **8(9)** (1978), 747–752.

[12] Ravsky, O. V., Paratopological groups I, *Mattematychni. Studii,* **16** (2001), 37–48.

[13] Ravsky, O. V., Pseudocompact paratopological groups that are topological, arXiv:1003.5343v3, Apr 7, 2012; http://arxiv.org/abs/1003.5343.

[14] Reznichenko, E. A., Extensions of functions defined on products of pseudocompact spaces and continuity of the inverse in pseudocompact groups, *Topology Appl.,* **59** (1994), 233–244.

[15] Romaguera, S. and Sanchis, M., Continuity of the inverse in pseudocompact paratopological groups, *Algebra Colloq.,* **14** (2007), 167–175.

[16] Sánchez, I., Subgroups of products of paratopological groups, *Topology Appl.,* **163** (2014), 160–173.

[17] Tkachenko, M., Embedding paratopological groups into topological products, *Topology Appl.,* **156** (2009), 1298–1305.

[18] Tkachenko, M., *Paratoplogical and semitopological groups vs topological groups,* in: K. P. Hart, J. van Mill and P. Simon (Eds.), Recent Progress in General Topology III, 2013, pp. 803–859.