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Some Notes on Closed Sequence-Covering Maps

Fucai LIN[1,*], **Shou LIN**[2]
1. *School of Mathematics and Statistics, Minnan Normal University, Fujian* 363000*, P. R. China;*
2. *Institute of Mathematics, Ningde Teachers' College, Fujian* 352100*, P. R. China*

Abstract In this paper, we mainly discuss the images of certain spaces under closed sequence-covering maps. It is showed that the property with a locally countable weak base is preserved by closed sequence-covering maps. And the following question is discussed: Are the closed sequence-covering images of spaces with a point-countable *sn*-network *sn*-first countable?

Keywords closed maps; sequence-covering maps; weak bases; *sn*-networks; *cs*-networks; *k*-semistratifiable spaces.

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1. Introduction

In this paper all spaces are T_1 and regular, all maps are continuous and onto. Yan, Lin and Jiang in [20] proved that metrizability is preserved by closed sequence-covering maps. Lin and Liu in [10] and [13] showed respectively that g-metrizability and sn-metrizability are also preserved by closed sequence-covering maps. In a recent paper, Liu, Lin and Ludwig [15] have proved that the property with a σ-compact-finite weak base is preserved by closed sequence-covering maps. Hence what kind of properties of spaces are preserved by closed sequence-covering mappings is an interesting problem. In this paper, we shall prove that some kinds of properties of spaces are preserved by closed sequence-covering maps, and also discuss the relation between spaces with a σ-point-discrete sn-network and spaces with a σ-point-discrete cs-network.

By $\mathbb{N}$, we denote the set of positive integers. Let $\tau(X)$ be the topology of a space X.

Let X be a space, and $P \subset X$. The set P is a sequential neighborhood of x in X if every sequence converging to x is eventually in P. The set P is a sequentially open subset of X if P is a sequential neighborhood of x in X for each $x \in P$. P is a sequentially closed subset of X if $X \setminus P$ is a sequentially open subset of X. The space X is said to be a sequential space [3] if each sequentially open subset is open in X.

Definition 1.1 *Let* $\mathcal{P} = \bigcup_{x\in X} \mathcal{P}_x$ *be a cover of a space* X *such that for each* $x \in X$, *(a) if*

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* Corresponding author
E-mail address: linfucai2008@aliyun.com; lfc19791001@163.com (Fucai LIN); shoulin60@163.com (Shou LIN)

$U, V \in \mathcal{P}_x$, then $W \subset U \cap V$ for some $W \in \mathcal{P}_x$; (b) $\mathcal{P}_x$ is a network of x in X, i.e., $x \in \bigcap \mathcal{P}_x$, and if $x \in U$ with U open in X, then $P \subset U$ for some $P \in \mathcal{P}_x$.

(1) The family $\mathcal{P}$ is called an sn-network [7] for X if each element of $\mathcal{P}_x$ is a sequential neighborhood of x in X for each $x \in X$. The space X is called sn-first countable [9], if X has an sn-network $\mathcal{P}$ such that each $\mathcal{P}_x$ is countable.

(2) The family $\mathcal{P}$ is called a weak base [1] for X if whenever $G \subset X$ satisfying for each $x \in X$ there is $P \in \mathcal{P}_x$ with $P \subset G$, G is open in X. The space X is called weakly first countable [1] or g-first countable [19], if X has a weak base $\mathcal{P}$ such that each $\mathcal{P}_x$ is countable.

A related concept for sn-networks is cs-networks.

Definition 1.2 *Let $\mathcal{P}$ be a family of subsets of a space X.*

(1) The family $\mathcal{P}$ is called a cs-network [5] for X if whenever a sequence $\{x_n\}_n$ converges to $x \in U \in \tau(X)$, there exist $m \in \mathbb{N}$ and $P \in \mathcal{P}$ such that $\{x\} \cup \{x_n : n \geqslant m\} \subset P \subset U$.

(2) The family $\mathcal{P}$ is called a k-network [17] for X if whenever K is a compact subset of X and $K \subset U \in \tau(X)$, there is a finite $\mathcal{P}' \subset \mathcal{P}$ such that $K \subset \cup\mathcal{P}' \subset U$.

(3) The family $\mathcal{P}$ is called a wcs^*-network [11] for X if whenever sequence $\{x_n\}_n$ converges to $x \in U \in \tau(X)$, there are a $P \in \mathcal{P}$ and a subsequence $\{x_{n_i}\}_i$ of $\{x_n\}_n$ such that $P \subset U$ and $x_{n_i} \in P$ for each $i \in \mathbb{N}$.

It is easy to see that [9]

(i) g-first countable spaces $\Leftrightarrow$ sn-first countable spaces and sequential spaces;

(ii) For a space X, weak bases $\Rightarrow$ sn-networks $\Rightarrow$ cs-networks $\Rightarrow$ wcs^*-networks, and k-networks $\Rightarrow$ wcs^*-networks;

(iii) For a sequential space X, sn-networks $\Rightarrow$ weak bases.

Definition 1.3 *Let $f : X \to Y$ be a map. Recall that f is a sequence-covering map [18] if whenever $\{y_n\}_n$ is a convergent sequence in Y, there is a convergent sequence $\{x_n\}_n$ in X with each $x_n \in f^{-1}(y_n)$.*

Definition 1.4 *Let (X, τ) be a topological space. We define a sequential closure-topology σ_τ [3] on X as follows: $O \in \sigma_\tau$ if and only if O is a sequentially open subset in (X, τ). The topological space (X, σ_τ) is denoted by σX.*

Readers may refer to [2, 4] for unstated definitions and terminologies.

2. Spaces with locally countable weak bases

Firstly, we prove that the property with a locally countable weak base is preserved by closed sequence-covering maps. A family $\mathcal{P}$ of subsets of a space X is called locally countable if each point at X has a neighborhood which intersects at most countably many elements of $\mathcal{P}$.

Let $f : X \to Y$ be a map. The map f is said to be boundary-compact if $\partial f^{-1}(y)$ is compact in X for each $y \in Y$.

Lemma 2.1 *The property with a locally countable k-network is preserved by closed boundary-*

compact maps.

Proof Let $f : X \to Y$ be a closed boundary-compact map, where X has a locally countable k-network $\mathcal{P}$. Since k-networks are hereditary with respect to closed subsets, we can suppose that f is a perfect map by [9, Lemma 1.3.2]. Thus $f(\mathcal{P}) = \{f(P) : P \in \mathcal{P}\}$ is a locally countable k-network for Y. $\Box$

The sequential fan S_ω is a space which is the quotient space by identifying all limit points of the topological sum of ω many convergent sequences. Every sn-first countable space contains no closed copy of S_ω.

Theorem 2.2 *The property with a locally countable weak base is preserved by closed sequence-covering maps.*

Proof Let $f : X \to Y$ be a closed sequence-covering map, where X has a locally countable weak base. By [12, Lemma 3.1], Y is g-first countable. Thus Y contains no closed copy of S_ω. So f is a boundary-compact map by [12, Lemma 3.2]. By Lemma 2.1, Y has a locally countable k-network. Therefore, Y has a locally countable weak base by [14, Theorem 2.1]. $\Box$

In the proof of Theorem 2.2, the space X is paracompact. In fact, since X has a locally countable weak base, X is a topological sum of spaces with countable weak bases [14], thus X is paracompact. So is Y.

However, closed sequence-covering maps do not preserve the property with a locally countable sn-network.

Example 2.3 For each $\alpha < \omega_1$, let X_α be a subspace $\{p\} \cup \mathbb{N}$ of $\beta\mathbb{N}$, where $p \in \beta\mathbb{N} \setminus \mathbb{N}$. Since X_α has no non-trivial convergent sequence, it has a countable sn-network. Put $X = \bigoplus_{\alpha<\omega_1} X_\alpha$, and let A be the set of all accumulative points of X. Obviously, the space X has a locally countable sn-network, and A is a closed subset of X. Take $Y = X/A$ and let $f : X \to Y$ be the natural quotient map. It follows that f is a closed map. Since Y has no non-trivial convergent sequence, the map f is also a sequence-covering map. It is easy to see that Y is not a locally countable space. Hence Y does not have a locally countable sn-network.

Although closed sequence-covering maps do not preserve the property with a locally countable sn-network, we have the following Theorem 2.6.

Lemma 2.4 *Let $\mathcal{P}$ be an sn-network for an sn-first countable space X. Then $\mathcal{P}$ is a weak base for σX.*

Proof Since σX is a sequential space, it suffices to prove that $\mathcal{P}$ is an sn-network for σX. Let $\mathcal{P} = \bigcup_{x\in X} \mathcal{P}_x$ as Definition 1.1, where each $\mathcal{P}_x$ is countable. We prove that $\mathcal{P}_x$ is a network at point $x \in \sigma X$. Without loss of generality, we can assume that $\mathcal{P}_x = \{P_i\}_{i\in\mathbb{N}}$ be a decreasing sequence. If $x \in U$ with U sequentially open in X, then $P_i \subset U$ for some $i \in \mathbb{N}$. Otherwise, there exists $x_i \in P_i \setminus U$ for each $i \in \mathbb{N}$. Then $x_i \to x$ as $i \to \infty$ in X. This is a contradiction with $x_i \notin U$. It is easy to see that each $P \in \mathcal{P}_x$ is a sequential neighborhood at point x in σX.

Hence, the family $\mathcal{P}$ is an sn-network for σX. $\Box$

Lemma 2.5 *Let $f : X \to Y$ be a closed sequence-covering map, where each singleton of X is a G_δ-set. Then $f : \sigma X \to \sigma Y$ is a closed sequence-covering map.*

Proof (1) The map f is continuous from σX to σY.

Let U be sequentially open in Y. We claim that $f^{-1}(U)$ is sequentially open in X. Suppose not, there exist a point $x \in f^{-1}(U)$ and a sequence $\{x_n\}_n$ in X such that each $x_n \notin f^{-1}(U)$ and $x_n \to x$. Since f is continuous from X to Y, $f(x_n) \to f(x) \in U$. This is a contradiction.

(2) The map f is a closed map from σX to σY.

Let A be sequentially closed in X. If $f(A)$ is not sequentially closed in Y, there exists a sequence $\{y_n\}_n \subset f(A)$ such that $\{y_n\}_n$ converges to $y \notin f(A)$. Without loss of generality, we can assume $y_n \neq y_m$ when $n \neq m$. Put $K = \{y_n : n \in \mathbb{N}\} \cup \{y\}$. For each $n \in \mathbb{N}$, choose $x_n \in f^{-1}(y_n) \cap A$, then $\{x_n\}_n$ is a sequence in $f^{-1}(K)$. Hence there exists a convergent subsequence $\{x_{n_k}\}_k$ of $\{x_n\}_n$ by [11, Lemma 2(b)], say $x_{n_k} \to x$. Then $x \in f^{-1}(y)$, and $x \notin A$ by $y \notin f(A)$. So $\{x_{n_k}\}_k$ is eventually in $X - A$ because A is sequentially closed. However, all $x_n \in A$. This is a contradiction.

(3) Since a space Z and its sequentially closure-space σZ have identically convergent sequence, it follows that f is a sequence-covering map from σX to σY. $\Box$

A family $\mathcal{P}$ of subsets of a space X is called star-countable if, for every $P \in \mathcal{P}$, P intersects at most countably many members of $\mathcal{P}$.

Theorem 2.6 *Let $f : X \to Y$ be a closed sequence-covering map, where X has a locally countable sn-network and σX is a regular space. Then Y has a star-countable sn-network.*

Proof Since each singleton of X is a G_δ-set, it follows that $f : \sigma X \to \sigma Y$ is a closed sequence-covering map by Lemma 2.5. Then σX has a locally countable weak base by Lemma 2.4. Since σX is regular, it follows from Theorem 2.2 that σY has a locally countable weak base. It is easy to see that σY has a star-countable sn-network $\mathcal{P}$. Obviously, the family $\mathcal{P}$ is an sn-network of Y. $\Box$

It is well known that spaces with a locally countable sn-network have a star-countable sn-network. However, there is a compact space with a star-countable sn-network, which has no locally countable sn-network. In fact, let X be the Stone-Čech compactification $\beta\mathbb{N}$ of $\mathbb{N}$. It is easy to see that X has a star-countable sn-network $\{\{x\} : x \in X\}$. But X does not have a locally countable sn-network.

3. Spaces with point-countable sn-networks

Liu has proved that the closed sequence-covering images of spaces with a point-countable weak base are g-first countable [12]. The following question is interesting.

Question 3.1 Let $f : X \to Y$ be a closed sequence-covering map, where X has a point-countable sn-network. Is Y sn-first countable?

Theorem 2.6 is a partial answer to this question. In this section we shall give other partial answers to the question.

Definition 3.2 *A space X is said to be a k-semistratifiable space [16] if for every $U \in \tau(X)$ there exists a sequence $\{F(n,U)\}_{n\in\mathbb{N}}$ of closed subsets of X such that*

(1) $U = \bigcup_{n\in\mathbb{N}} F(n,U)$;

(2) If $V \subset U$, then $F(n,V) \subset F(n,U)$;

(3) If a compact subset $K \subset U$, then $K \subset F(m,U)$ for some $m \in \mathbb{N}$.

Let $\mathcal{P}$ be a family of subsets of space X. The family $\mathcal{P}$ is s-closure-preserving [10] in X if $\cup\mathcal{P}'$ is a sequentially closed subset in X for every $\mathcal{P}' \subset \mathcal{P}$. The family $\mathcal{P}$ is s-discrete [10] in X if $\mathcal{P}$ is disjoint and s-closure-preserving in X. A subset D of X is s-discrete if $\{\{x\} : x \in D\}$ is s-discrete in X.

Theorem 3.3 *Let $f : X \to Y$ be a closed sequence-covering map, where X has a point-countable sn-network. If X satisfies one of the following conditions, then Y is an sn-first countable space.*

(1) Each singleton of X is a G_δ-set and σX is regular;

(2) X is a k-semistratifiable space.

Proof The space σX has a point-countable weak base by Lemma 2.4. We only need to prove that σY is g-first countable by Definition 1.1.

(1) If X satisfies the conditions (1), then $f : \sigma X \to \sigma Y$ is a closed sequence-covering map by Lemma 2.5. Hence σY is g-first countable by [12, Lemma 3.1].

(2) If X is a k-semistratifiable space, then each singleton of X is a G_δ-set, thus $f : \sigma X \to \sigma Y$ is also a closed sequence-covering map. By [9, Lemma 2.1.6 and Theorem 2.2.5], the space Y has a point-countable k-network. Suppose σY is not g-first countable, then σY contains a closed copy of S_ω by [9, Theorem 2.1.9]. Let $\{y\} \cup \{y_i(n) : i \in \mathbb{N}, n \in \mathbb{N}\}$ be a closed copy of S_ω in σY, here $y_i(n) \to y$ as $i \to \infty$. For every $k \in \mathbb{N}$, put $L_k = \cup\{y_i(n) : i \in \mathbb{N}, n \le k\}$. Hence L_k is a sequence converging to y. Let M_k be a sequence of σX converging to $u_k \in f^{-1}(y)$ such that $f(M_k) = L_k$, we rewrite $M_k = \cup\{x_i(n,k) : i \in \mathbb{N}, n \le k\}$ with each $f(x_i(n,k)) = y_i(n)$.

Case 1 The set $\{u_k : k \in \mathbb{N}\}$ is finite.

There are a $k_0 \in \mathbb{N}$ and an infinite subset $\mathbb{N}_1 \subset \mathbb{N}$ such that $M_k \to u_{k_0}$ for every $k \in \mathbb{N}_1$, then σX contains a closed copy of S_ω. Hence σX is not g-first countable. This is a contradiction.

Case 2 The set $\{u_k : k \in \mathbb{N}\}$ has a non-trivial convergent sequence in σX.

Without loss of generality, we suppose that $u_k \to u$ as $k \to \infty$. Since each singleton of X is a G_δ-set, let $\{U_m\}_m$ be a sequence of open subsets of X with $\overline{U}_{m+1} \subset U_m$, and $\bigcap_{m\in\mathbb{N}} U_m = \{u\}$. Fix n, pick $x_{i_m}(n,k_m) \in U_m \cap \{x_i(n,k_m)\}_{i\in\mathbb{N}}$. We can suppose that $i_m < i_{m+1}$. Then $\{f(x_{i_m}(n,k_m))\}_m$ is a subsequence of $\{y_i(n)\}_i$. Since f is closed, $\{x_{i_m}(n,k_m)\}_m$ is not discrete in σX. Then there is a sequence of $\{x_{i_m}(n,k_m)\}_m$ converging to a point $b \in X$ because σX is a sequential space. It is easy to see that $b = u$ by $x_{i_m}(n,k_m) \in U_m$ for every $m \in \mathbb{N}$. Hence $x_{i_m}(n,k_m) \to u$ as $m \to \infty$. Then $\{u\} \cup \{x_{i_m}(n,k_m) : n \in \mathbb{N}, m \in \mathbb{N}\}$ is a closed copy of S_ω in

σX. Thus, σX is not g-first countable. This is a contradiction.

Case 3 The set $\{u_k : k \in \mathbb{N}\}$ is discrete in σX.

Since $\{u_k : k \in \mathbb{N}\}$ is discrete in σX, $\{u_k : k \in \mathbb{N}\}$ is s-discrete in X. By [10, Lemma 1.3], since X is a k-semistratifiable space, there exists an s-disctete extension of sequential neighborhoods $\{V_k\}_{k\in\mathbb{N}}$ in X such that $u_k \in V_k$ for each $k \in \mathbb{N}$. It is obvious that $\{V_k\}_k$ is discrete in σX. Pick $x_{i_k}(1,k) \in V_k \cap \{x_i(1,k)\}_i$ such that $\{f(x_{i_k}(1,k))\}_k$ is a subsequence of $\{y_i(n)\}_i$. Since $\{x_{i_k}(1,k)\}_k$ is discrete in σX, $\{f(x_{i_k}(1,k))\}_k$ is discrete in σY. This is a contradiction.

In a word, the space σY is g-first countable. Hence Y is an sn-first countable space. □

Next, we discuss a special space with a point-countable sn-network.

Definition 3.4 *Let $\mathcal{B} = \{B_\alpha : \alpha \in H\}$ be a family of subsets of a space X. The family $\mathcal{B}$ is point-discrete if $\{x_\alpha : \alpha \in H\}$ is closed discrete in X, whenever $x_\alpha \in B_\alpha$ for each $\alpha \in H$.*

In [6], Lin and Shen posed the following question.

Question 3.5 Is the property with a σ-point-discrete sn-network preserved by closed sequence-covering maps?

In [6], Lin and Shen have proved that a space X has a σ-point-discrete sn-network if and only if X is an sn-first countable space with a σ-point-discrete cs-network. Recently, Liu posed the following question in a private communication with the authors.

Question 3.6 If X is an α_4-space with a σ-point-discrete cs-network, has X a σ-point discrete sn-network?

It is known [9] that for a space X, X is sn-first countable $\Rightarrow$ X is an α_4-space $\Leftrightarrow$ σX is an α_4-space $\Rightarrow$ σX contains no closed copy of S_ω $\Rightarrow$ X contains no closed copy of S_ω. Next we shall give an affirmative answer to Question 3.6, and a partial answer to Question 3.5.

A family $\mathcal{P}$ of subsets of a space X is called compact-finite if, each compact subset of X intersects at most finitely many members of $\mathcal{P}$.

Lemma 3.7 *Every space with a σ-point-discrete wcs^*-network has a σ-compact-finite wcs^*-network.*

Proof Let $\mathcal{P}$ be a σ-point-discrete wcs^*-network. Denote $\mathcal{P}$ by $\bigcup_{n\in\mathbb{N}} \mathcal{P}_n$, where each $\mathcal{P}_n$ is point-discrete in X. For each $n \in \mathbb{N}$, put $D_n = \{x \in X : \mathcal{P}_n$ is not point-finite at $x\}$, and let $\mathcal{F}_n = \{P \setminus D_n : P \in \mathcal{P}_n\} \cup \{\{x\} : x \in D_n\}$. Then $\mathcal{F}_n$ is compact-finite in X by [9, (3.1) in Lemma 4.1.3].

If a sequence $\{x_n\}_n$ converges to a point $x \in U \in \tau(X)$, there are a $P \in \mathcal{P}$ and a subsequence $\{x_{n_i}\}_i$ of $\{x_n\}_n$ such that $P \subset U$ and $x_{n_i} \in P$ for each $i \in \mathbb{N}$. Then $P \in \mathcal{P}_m$ for some $m \in \mathbb{N}$. We can assume the sequence $\{x_{n_i}\}_i$ is non-trivial. Since $\{x\} \cup \{x_{n_i} : i \in \mathbb{N}\}$ is compact, $D_m \cap (\{x\} \cup \{x_{n_i} : i \in \mathbb{N}\})$ is finite. There is $i_0 \in \mathbb{N}$ such that $x_{n_i} \notin D_m$ for each $i \geq i_0$, and $x_{n_i} \in P \setminus D_m \subset U$. Therefore, $\bigcup_{n\in\mathbb{N}} \mathcal{F}_n$ is a σ-compact-finite wcs^*-network for X. □

Theorem 3.8 *The following are equivalent for a space X:*

(1) X has a σ-point-discrete sn-network;

(2) X is an sn-first countable space with a σ-point-discrete cs-network;

(3) X is an α_4-space with a σ-point-discrete cs-network;

(4) X has a σ-point-discrete cs-network and σX contains no closed copy of S_ω.

Proof (1) $\Leftrightarrow$ (2) by [6, Theorem 2.2]. (2) $\Rightarrow$ (3) $\Rightarrow$ (4) is obvious [9]. Since a space X with a point-countable wcs^*-network is sn-first countable if σX contains no closed copy of S_ω [9, Theorem 2.1.9], (4) $\Rightarrow$ (2) by Lemma 3.7. □

We cannot replace the condition "σX contains no closed copy of S_ω" by "X contains no closed copy of S_ω" in (4) of Theorem 3.8. In fact, the space T in [8, Example 3.19] has a countable cs-network and contains no copy of S_ω. But T is not sn-first countable.

Finally, we shall give a partial answer to Question 3.5.

Theorem 3.9 *Let $f : X \to Y$ be a closed sequence-covering map, where X has a σ-point-discrete sn-network. If X satisfies one of the following conditions, then Y has a σ-point-discrete sn-network.*

(1) Each singleton of X is a G_δ-set and σX is regular;

(2) X is a k-semistratifiable space.

Proof Obviously, the property with a σ-point-discrete cs-network is preserved by closed sequence-covering maps.

(1) If X satisfies the conditions (1), then Y is sn-first countable by Theorem 3.9 and [6, Theorem 2.1]. Hence, the space Y has a σ-point-discrete sn-network by Theorem 3.8.

(2) Let X be a k-semistratifiable space. Since X is an α_4-space, it follows that Y is an α_4-space by [10, Theorem 2.1]. Hence, the space Y has a σ-point-discrete sn-network by Theorem 3.8. □

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