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A NOTE ON *g*-DEVELOPABLE SPACES

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Abstract

In this paper, we give some characterizations of g-developable spaces, which prove that a space is g-developable if and only if it has a weak-development consisting of *cs*-covers (*sn*-covers), or it is a strong compact-covering, quotient π-images of a metric space.

1. Introduction and Definitions

In 1976, Lee [7] introduced the concept of g-developable spaces as a generalization of developable spaces, and obtained the following:

(1) A Hausdorff space is developable if and only if it is Fréchet and g-developable.

(2) A Hausdorff space is g-developable if and only if it is Cauchy.

(3) A Hausdorff g-developable space is a quotient π-image of a metric space.

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In 1991, Tanaka showed that a Hausdorff space is weak Cauchy if and only if it is a quotient π-image of a metric space, and gave an example explaining weak Cauchy is not necessarily Cauchy.

In this paper, we further discuss a g-developable space, give its "development" characterizations by using of weak-developments, *cs*-covers and *sn*-covers, and prove that a space is g-developable if and only if it is a strong compact-covering, quotient π-image of a metric space, which generalize the result of Lee and Tanaka.

In this paper, all spaces are Hausdorff, all maps are continuous and surjective. N denotes the set of all natural numbers. $\tau(X)$ denotes the topology on X. For a collection $\mathcal{P}$ of subsets of a space X and a map $f : X \to Y$, denote $\{f(P) : P \in \mathcal{P}\}$ by $f(\mathcal{P})$. For the usual product space $\prod_{i \in N} X_i$, π_i denotes the projection of $\prod_{i \in N} X_i$ onto X_i. For a sequence $\{x_n\}$ in X, denote $\langle x_n \rangle = \{x_n : n \in N\}$.

Definition 1.1. Let $f : X \to Y$ be a map. Then

(1) f is called a *compact-covering* [12] (respectively, *pseudo-sequence-covering* [5]) *map*, if each compact subset (respectively convergent sequence including its limit point) of Y is the image of some compact subset of X.

(2) f is a *sequence-covering map* [15], if whenever $\{y_n\}$ is a convergent sequence in Y, then there exists a convergent sequence $\{x_n\}$ in X such that each $x_n \in f^{-1}(y_n)$.

f is called *strong compact-covering*, if it is both a compact-covering and a sequence-covering.

(3) f is called a *π-map* [14], if (X, d) is a metric space, and for each $y \in Y$ and its open neighborhood V in Y, $d(f^{-1}(y), M \setminus f^{-1}(V)) > 0$.

Definition 1.2 [3]**.** Let X be a space, and $P \subset X$. Then

(1) a sequence $\{x_n\}$ in X is called *eventually* in P, if $\{x_n\}$ converges to x, and there exists $m \in N$ such that $\{x\} \cup \{x_n : n \geq m\} \subset P$.

(2) P is called a *sequential neighborhood* of x in X, if whenever a sequence $\{x_n\}$ in X converges to x, then $\{x_n\}$ is eventually in P.

(3) X is called a *sequential space*, if any $A \subset X$ which is a sequential neighborhood of each of its points is open in X.

Definition 1.3 [10]**.** Let $\mathcal{P}$ be a collection of subsets of a space X. Then

(1) $\mathcal{P}$ is called a *cs-cover* for X, if $\mathcal{P}$ is a cover for X, and every convergent sequence in X is eventually in some element of $\mathcal{P}$.

(2) $\mathcal{P}$ is called an *sn-cover* for X, if $\mathcal{P}$ is a cover for X, every element of $\mathcal{P}$ is a sequential neighborhood of some point in X, and for each $x \in X$ there exists a sequential neighborhood P of x in X such that $P \in \mathcal{P}$.

(3) $\mathcal{P}$ is called a *cfp* (i.e., *compact finite partition*) *cover* of a compact subset K in X, if there is a finite collection $\{K_\alpha : \alpha \in J\}$ of closed subsets of K and $\{P_\alpha : \alpha \in J\} \subset \mathcal{P}$ such that $K = \bigcup\{K_\alpha : \alpha \in J\}$ and each $K_\alpha \subset P_\alpha$.

$\mathcal{P}$ is called a *cfp-cover* for X, if $\mathcal{P}$ is a cover for X, and for any compact subset K of X, there exists a finite subcollection $\mathcal{P}^* \subset \mathcal{P}$ such that $\mathcal{P}^*$ is a *cfp* cover of K in X.

Definition 1.4. Let $\{\mathcal{P}_n\}$ be a sequence of covers of a space X.

(1) $\{\mathcal{P}_n\}$ is called a *point-star network* for X, if for each $x \in X$, $\langle st(x, \mathcal{P}_n)\rangle$ is a network of x in X.

(2) $\{\mathcal{P}_n\}$ is called a *weak-development* for X, if for each $x \in X$, $\langle st(x, \mathcal{P}_n)\rangle$ is a weak neighborhood base for X.

Definition 1.5 [1]**.** Let (X, d) be a symmetrizable space. Then

(1) a sequence $\{x_n\}$ in X is called *d-Cauchy*, if for each $\varepsilon > 0$, there exists $k \in N$ such that $d(x_m, x_n) < \varepsilon$ for all $n, m > k$.

(2) X is called *Cauchy*, if each convergent sequence in X is d-Cauchy.

For a space X, let g be a map defined on $N \times X$ to the power-set of X such that $x \in g(n, x)$ and $g(n+1, x) \subset g(n, x)$ for each $n \in N$ and $x \in X$, a subset U of X is open if for each $x \in U$, there exists $n \in N$ such that $g(n, x) \subset U$. We call such a map a *CWC-map* (i.e., *countable weakly-open covering map*).

Definition 1.6 [7]**.** A space X is called *g-developable*, if X has a *CWC*-map g with the following property: If $x, x_n \in g(n, g_n)$ for each $n \in N$, then sequence $\{x_n\}$ converges to x.

2. Results

Theorem 2.1. *The following are equivalent for a space X*:

(1) *X is a g-developable space*.

(2) *X is a Cauchy space*.

(3) *X has a weak-development consisting of cs-covers*.

(4) *X has a weak-development consisting of sn-covers*.

(5) *X is a strong compact-covering*, *quotient* π-*image of a metric space*.

(6) *X is a sequence-covering*, *quotient* π-*image of a metric space*.

Proof. (1) $\Leftrightarrow$ (2) follows from Theorem 2.3 in [7].

(2) $\Rightarrow$ (3) Suppose X is a Cauchy space. For each $n \in N$, put

$$\mathcal{P}_n = \{A \subset X : \sup\{d(x, y) : x, y \in A\} < 1/n\}$$

then $st(x, \mathcal{P}_n) = B(x, 1/n)$ for each $x \in X$, so $\{\mathcal{P}_n\}$ is a point-star network for X.

For each sequence $\{x_n\}$ converging to $x \in X$, since $\{x_n\}$ is d-Cauchy and X is symmetrizable, then there exists $m \in N$ such that $d(x, x_i) < 1/(n+1)$ and $d(x_i, x_j) < 1/(n+1)$ for all $i, j \geq m$ by Lemma 9.3 in [4]. Put

$$P = \{x\} \cup \{x_i : i \geq m\}$$

then $P \in \mathcal{P}_n$. Hence each $\mathcal{P}_n$ is a cs-cover for X.

Obviously, X is a sequential space. For each $x \in X$ and $n \in N$, since $\mathcal{P}_n$ is a cs-cover for X, then $st(x, \mathcal{P}_n)$ is a sequential neighborhood of x in X. So $\langle st(x, \mathcal{P}_k)\rangle$ is a weak neighborhood base of x in X. Thus, $\{\mathcal{P}_n\}$ is a weak-development for X.

(3) $\Rightarrow$ (4) Suppose $\{\mathcal{P}_n\}$ is a cs-cover weak-development for X. We can assume that $\mathcal{P}_{n+1}$ refines $\mathcal{P}_n$ for each $n \in N$. For each $x, y \in X$, denotes

$$t(x, y) = \min\{n : x \notin st(y, \mathcal{P}_n)\}\,(x \neq y).$$

We define

$$d(x, y) = \begin{cases} 0, & x = y, \\ 2^{-t(x, y)}, & x \neq y, \end{cases}$$

then $d : X \times X \to [0, +\infty)$ is a symmetric on X.

Claim. For each $x, y \in X$, $x \in st(y, \mathcal{P}_n)$ if and only if $t(x, y) > n$.

In fact, the if part is obvious. The only if part: Suppose $x \in st(y, \mathcal{P}_n)$ but $t(x, y) \leq n$, since $\mathcal{P}_n$ refine $\mathcal{P}_{t(x, y)}$, $st(y, \mathcal{P}_n) \subset st(y, \mathcal{P}_{t(x, y)})$. Note that $x \notin st(y, \mathcal{P}_{t(x, y)})$, so $x \notin st(y, \mathcal{P}_n)$, a contradiction.

For each $x \in X$ and $n \in N$, $st(x, \mathcal{P}_n) = B(x, 1/2^n)$ by the Claim. Because $\{\mathcal{P}_n\}$ is a point-star network for X, then (X, d) is symmetrizable. And d has the following property: for each $x \in X$ and $\varepsilon > 0$, there exists $\delta = \delta(x, \varepsilon) > 0$ such that $d(x, y) < \delta$ and $d(x, z) < \delta$ imply $d(y, z) < \varepsilon$. Otherwise, there exist $\varepsilon_0 > 0$ and two sequences $\{y_n\}$ and $\{z_n\}$ in X such that $d(y_n, z_n) \geq \varepsilon_0$ whenever $d(x, y_n) < 1/2^n$ and $d(x, z_n) < 1/2^n$. From $\mathcal{P}_n$ is a point-star network for X, $\{y_n\}$ and $\{z_n\}$ all converge to x. We choose $k \in N$ such that $1/2^k < \varepsilon_0$. Since $\mathcal{P}_k$ is a cs-cover for X, $\{y_m, z_m\} \subset P$ for some $m \in N$ and $P \in \mathcal{P}_k$. Thus $y_m \in st(z_m, \mathcal{P}_k)$. By the Claim, $t(y_m, z_m) > k$. Thus, $d(y_m, z_m) = 1/2^{t(y_m, z_m)} < 1/2^k < \varepsilon_0$, a contradiction.

For each $x \in X$ and $n \in N$, we can pick $\delta = \delta(x, n)$ such that $d(y, z) < 1/n$ whenever $d(x, y) < \delta$ and $d(x, z) < \delta$. Let $g(n, x) = B(x, \delta(x, n))$.

Since $\mathcal{P}_n$ is a cs-cover for X, $st(x, \mathcal{P}_n)$ is a sequential neighborhood of x in X, so $g(n, x)$ is also. Put

$$\mathcal{F}_n = \{g(n, x) : x \in X\},$$

then every $\mathcal{F}_n$ is a sn-cover for X.

If $\{\mathcal{F}_n\}$ is not a point-star network for X, then there exist $x \in G \in \tau(X)$ and two sequences $\{x_n\}$ and $\{y_n\}$ in X such that $x \in g(n, y_n)$ and $x_n \in g(n, y_n) \setminus G$. So $\{x_n\}$ does not converge to x, and $d(y_n, x) < \delta(y_n, n)$, $d(y_n, x_n) < \delta(y_n, n)$. By the condition, $d(x, x_n) < 1/n$. This implies that $\{x_n\}$ converges x, a contradiction. Hence is a point-start network for X.

Obviously, X is a sequential space. Since $st(x, \mathcal{F}_n)$ is a sequential neighborhood of x in X for each $x \in X$ and $n \in N$, $\{\mathcal{F}_n\}$ is a weak-development for X.

(3) $\Rightarrow$ (2) Suppose $\{\mathcal{P}_i\}$ is a weak-development consisting of cs-covers for X. We can assume that $\mathcal{P}_{n+1}$ refines $\mathcal{P}_n$ for each $n \in N$. A similar proof of (3) $\Rightarrow$ (4), we can define a symmetric d on X such that $st(x, \mathcal{P}_n) = B(x, 1/2^n)$ for each $x \in X$ and $n \in N$. So (X, d) is symmetrizable. For each sequence $\{x_n\}$ in X converging to $x \in X$ and $\varepsilon > 0$, there exists $k \in N$ such that $1/2^k < \varepsilon$. Since $\mathcal{P}_k$ is a cs-cover for X, there exist $P \in \mathcal{P}_k$ and $l \in N$ such that $\{x\} \cup \{x_n : n \geq l\} \subset P$. If $n, m \geq l$, then $x_n, x_m \in P$, so $x_n \in st(x_m, \mathcal{P}_k)$. Thus $t(x_n, x_m) > k$ by the Claim in (3) $\Rightarrow$ (4). Hence $d(x_n, x_m) = 1/2^{t(x_n, x_m)} < 1/2^k < \varepsilon$ whenever $n, m \geq l$. Therefore $\{x_n\}$ is d-Cauchy. This implies that X is Cauchy.

(4) $\Rightarrow$ (5) Suppose $\{\mathcal{P}_n\}$ is a weak-development consisting of sn-covers for X. For each $i \in N$, let $\mathcal{P}_i = \{P_\alpha : \alpha \in \Lambda_i\}$, endow Λ_i with the discrete topology, then Λ_i is a metric space. Put

$$M = \left\{\alpha = (\alpha_i) \in \prod_{i \in N} \Lambda_i : \langle P_{\alpha_i} \rangle \text{ forms a network at some point } x_\alpha \text{ in } X\right\}$$

and endow M with the subspace topology induced from the usual product topology of the collection $\{\Lambda_i : i \in N\}$ of metric spaces, then M is a metric space. Since X is Hausdorff, x_α is unique in X. For each $\alpha \in M$. We define $f : M \to X$ by $f(\alpha) = x_\alpha$. For each $x \in X$ and $i \in N$, there exists $\alpha_i \in \Lambda_i$ such that $x \in P_{\alpha_i}$. From $\{\mathcal{P}_i\}$ is a point-star network for X, $\{P_{\alpha_i} : i \in N\}$ is a network of x in X. Put $\alpha = (\alpha_i)$, then $\alpha \in M$ and $f(\alpha) = x$. Thus f is surjective. Suppose $\alpha = (\alpha_i) \in M$ and $f(\alpha) = x \in U \in \tau(X)$, then there exists $n \in N$ such that $P_{\alpha_n} \subset U$. Put

$$V = \{\beta \in M : \text{the } n\text{-th coordinate of } \beta \text{ is } \alpha_n\}$$

then $\alpha \in V \in \tau(X)$, and $f(V) \subset P_{\alpha_n} \subset U$. Hence f is continuous.

(1) f is a π-map. For each $\alpha, \beta \in M$, we define

$$d(\alpha, \beta) = \begin{cases} 0, & \alpha = \beta, \\ \max\{1/k : \pi_k(\alpha) \neq \pi_k(\beta)\}, & \alpha \neq \beta, \end{cases}$$

then d is a distance on M. Because the topology of M is the subspace topology induced from the usual product topology of the collection $\{\Lambda_i : i \in N\}$ of discrete spaces, thus d is metric on M. For each $x \in U \in \tau(X)$, note that $\{\mathcal{P}_n\}$ is a point-star network for X, there exists $n \in N$ such that $st(x, \mathcal{P}_n) \subset U$. For $\alpha \in f^{-1}(x)$, $\beta \in M$, if $d(\alpha, \beta) < 1/n$, then $\pi_i(\alpha) = \pi_i(\beta)$ for all $i \leq n$. So $x \in P_{\pi_n(\alpha)} = P_{\pi_n(\beta)}$. Thus

$$f(\beta) \in \bigcap_{i \in N} P_{\pi_i(\beta)} \subset P_{\pi_n(\beta)} \subset U.$$

Hence

$$d(f^{-1}(x), M \setminus f^{-1}(U)) \geq 1/n.$$

Therefore f is a π-map.

(2) f is a sequence-covering map.

Suppose $\{x_n\}$ converges to x in X. For each $i \in N$, since every $\mathcal{P}_i$ is a sn-cover for X, then there exists $\alpha_i \in \Lambda_i$ such that P_{α_i} is a sequential neighborhood of x in X, so $\{x_n\}$ is eventually in P_{α_i}. From $\{\mathcal{P}_i\}$ is a point

-star network for X, $\langle P_{\alpha_i}\rangle$ is a network of x in X. Put $\beta_x = (\alpha_i) \in \prod_{i\in N}\Lambda_i$, then $\beta_x \in f^{-1}(x)$. For each $n \in N$, if $x_n \in P_{\alpha_i}$, let $\alpha_{in} = \alpha_i$; if $x_n \notin P_{\alpha_i}$, pick $\alpha_{in} \in \Lambda_i$ such that $x_n \in P_{\alpha_{in}}$. Thus there exists $n_i \in N$ such that $\alpha_{in} = \alpha_i$ for all $n > n_i$. So $\{\alpha_{in}\}$ converges to α_i. For each $n \in N$, put

$$\beta_n = (\alpha_{in}) \in \prod_{i\in N}\Lambda_i,$$

then $\beta_n \in f^{-1}(x_n)$ and $\{\beta_n\}$ converges to β_x. Thus f is sequence-covering.

(3) f is a quotient map.

Since f is sequence-covering, by Proposition 2.1.16 in [8], f is quotient.

(4) f is a compact-covering map.

First, we prove that each $\mathcal{P}_n$ is a *cfp*-cover for X. As the proof of (3) $\Rightarrow$ (4), we can define $\rho : X \times X \to [0, +\infty)$, then ρ is a symmetric on X and (X, ρ) is symmetrizable. If K is compact in X, then subspace K is symmetrizable. Since compact symmetrizable space is metrizable (see [13]), subspace K is metrizable. For each $x \in K$, there exists $P_x \in \mathcal{P}_n$ such that P_x is a sequential neighborhood of x in X, then $x \in Int_K(P_x \cap K)$. Thus $\{Int_K(P_x \cap K) : x \in K\}$ is a open cover for subspace K, so there is a finite collection $\{K_i : i \leq l\}$ of closed subsets of K and $\{Int_K(P_{x_i} \cap K) : i \leq l\} \subset \mathcal{P}_n$ such that $K = \bigcup\{K_i : i \leq l\}$ and each $K_i \subset Int_K(P_{x_i} \cap K)$. Hence $\{P_{x_i} : i \leq l\}$ is a *cfp*-cover of K in X. This implies that each $\mathcal{P}_n$ is a *cfp*-cover for X.

Next, we prove that f is compact-covering. Suppose K is compact in X. From each $\mathcal{P}_n$ is a *cfp*-cover for X, there exists its finite subcollection $\mathcal{P}_n^K$ such that $\mathcal{P}_n^K$ is a *cfp*-cover of K in X. Thus there is a finite collection $\{K_\alpha : \alpha \in J_n\}$ of closed subsets of K and $\{P_\alpha : \alpha \in J_n\} \subset \mathcal{P}_n^K$ such that $K = \bigcup\{K_\alpha : \alpha \in J_n\}$ and each $K_\alpha \subset P_\alpha$. Obviously, each K_α is compact in X. Put

$$L = \left\{(\alpha_i) : \alpha_i \in J_i, \bigcap_{i \in N} K_{\alpha_i} \neq \varnothing\right\},$$

then

(i) L is compact in M.

In fact, $\forall (\alpha_i) \notin L$, $\bigcap_{i \in N} K_{\alpha_i} = \varnothing$. From $\bigcap_{i \in N} K_{\alpha_i} = \varnothing$, there exists $n_0 \in N$ such that $\bigcap_{i=1}^{n_0} K_{\alpha_i} = \varnothing$.

Put

$$W = \{(\beta_i) : \beta_i \in J_i, \beta_i = \alpha_i, 1 \leq i \leq n_0\},$$

then W is a open neighborhood of (α_i) in $\prod_{i \in N} J_i$, and $W \cap L = \varnothing$. Thus L is closed in $\prod_{i \in N} J_i$. By $\prod_{i \in N} J_i$ is compact in $\prod_{i \in N} \Lambda_i$, L is compact in M.

(ii) $L \subset M$, $f(L) = K$.

In fact, $\forall (\alpha_i) \in L$, $\bigcap_{i \in N} K_{\alpha_i} \neq \varnothing$. Pick $x \in \bigcap_{i \in N} K_{\alpha_i}$, then $\langle P_{\alpha_i} \rangle$ is a network of x in X, so $(\alpha_i) \in M$. This implies $L \subset M$.

$\forall x \in K$, for each $i \in N$, pick $\alpha_i \in J_i$ such that $x \in K_{\alpha_i}$. Thus $f((\alpha_i)) = x$, so $K \subset f(L)$. Obviously, $f(L) \subset K$. Hence $f(L) = K$.

In words, f is compact-covering.

(5) $\Rightarrow$ (6) is obvious.

(6) $\Rightarrow$ (3) Suppose X is a image of a metric space (M, d) under a sequence-covering, quotient π-map f. For each $n \in N$, put $\mathcal{B}_n = \{B(z, 1/n) : z \in M\}$ and $\mathcal{P}_n = f(\mathcal{B}_n)$, here $B(z, 1/n) = \{y \in M : d(z, y) < 1/n\}$. Then $\{\mathcal{P}_n\}$ is a point-star network for X. In fact, for each $x \in X$ and its open neighborhood U, since f is a π-map, there exists $n \in N$ such that $d(f^{-1}(x), M \setminus f^{-1}(U)) > 1/n$. We can pick $m \in N$ such that $m \geq 2n$. If

$z \in M$ with $x \in f(B(z, 1/m))$, then

$$f^{-1}(x) \bigcap B(z, 1/m) \neq \emptyset.$$

If $B(z, 1/m) \not\subset f^{-1}(U)$, then

$$d(f^{-1}(x), M \setminus f^{-1}(U)) \leq 2/m \leq 1/n,$$

a contradiction. Thus $B(z, 1/m) \subset f^{-1}(U)$, so $f(B(z, 1/m)) \subset U$. Hence $st(x, \mathcal{P}_m) \subset U$. This implies that $\{\mathcal{P}_n\}$ is a point-star network for X.

It is clear that X is a sequential space. We need only prove that each $\mathcal{P}_n$ is a cs-cover for X. For each $n \in N$, since $\mathcal{B}_n$ is a cs-cover for M and sequence-covering maps preserve cs-covers, $\mathcal{P}_n$ is a cs-cover for X.

Example 2.2. Let Z be the topological sum of the unite interval $[0, 1]$, and the collection $\{S(x) : x \in [0, 1]\}$ of 2^{ω} convergent sequence $S(x)$. Let X be the space obtained from Z by identifying the limit point of $S(x)$ with $x \in [0, 1]$, for each $x \in [0, 1]$. Then, from Example 2.9.27 in [8] or see Example 9.8 in [5], we have the following facts:

(1) X is a compact-covering, quotient compact image of a locally compact metric space.

(2) X has no point-countable cs-networks.

From the fact above, Theorem 1 in [9] and Theorem 2.1, the following holds:

A compact-covering, quotient π-image of a metric space is not a g-developable space.

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